EMAIL: ALESSIOR@KTH.SE

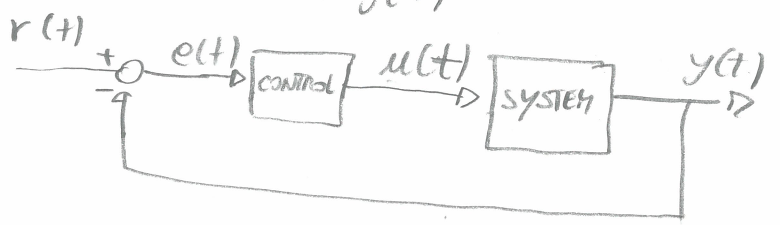
WEBSITE: RSSALESSIO.GITHUB.IO

PID CONTROL

- P: proportional
- I: integral
- D: derivative

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau)d\tau + K_D \dot{e}(t)$$

where $e(t) = r(t) - y(t)$



$$U(s) = \mathcal{L}\{u(t)\} = K_P E(s) + K_I \frac{E(s)}{s} + K_D sE(s)$$

$$E(s) = \mathcal{L}\{e(t)\}$$

It's a typical control strategy.

- **The proportional part**: may make the system faster, at the cost of stability. Does not help to remove steady state error (just reduce!). High K_P causes oscillations.
- **The integral part**: adds a pole in 0, but can remove steady state error.
- **Derivative part**: Predict change of error by using $\dot{e}(t) = \frac{d}{dt}e(t)$. Adds a zero. Makes the control action quicker.

EX 3.24a

A, B have steady state error $\Rightarrow$ no integral part. A is quicker than B, and has no oscillations. Hence, there is a zero in A $\Rightarrow$ it's (iii), and B is (i).

Similarly, C is (iv) and D is (ii).

EXAMPLE

$$G(s) = \frac{1}{s+1} \qquad U(s) = \underbrace{\left(K_P + K_I \frac{1}{s} + K_D s\right)}_{K(s)} E(s)$$

error, Reference

$$Y(s) = \underbrace{\frac{1}{s+1} K(s)}_{L(s)} E(s) = \frac{1}{s+1} K(s)\left(R(s) - Y(s)\right), \quad E(s) = R(s) - Y(s)$$

output

NB: $L(s) = G(s)C(s)$ is also called LOOP TRANSFER FUNCTION

$$\Rightarrow Y(s)\left(1 + G(s)K(s)\right) = G(s)K(s)R(s)$$

$$\Rightarrow Y(s) = \frac{G(s)K(s)}{1+G(s)K(s)} R(s)$$

new poles! when we close a feedback loop the poles will change! ZEROS do NOT CHANGE!!

The poles of the closed loop system are given by $1 + K(s)G(s) = 0$

In our case $1 + \frac{1}{s+1}\left(K_p + K_I \frac{1}{s} + K_d s\right) = 0$

$$s + 1 + K_p + K_I \frac{1}{s} + K_D s = 0$$

$$\downarrow$$

$$s(1 + K_D) + K_I \frac{1}{s} + (K_p + 1) = 0$$

$$\downarrow$$

$$s^2(1 + K_D) + (K_p + 1)s + K_I = 0$$

$$s^2 + \frac{(K_p+1)}{(K_D+1)} s + \frac{K_I}{(K_D+1)} = 0 \Rightarrow \text{solutions are}$$

$$s_{1,2} = \frac{-(K_p+1)/(K_D+1) \pm \sqrt{\left((K_p+1)/(K_D+1)\right)^2 - 4 K_I/(K_D+1)}}{2}$$

← NEW POLES.

STEP RESPONSE FOR 1ST ORDER SYSTEMS

$G(s) = \frac{\mu}{1+Ts} \Rightarrow y(t) = \mu(1 - e^{-t/T}),\ t \geq 0,$ when we apply a step signal to the system.

1) $y(0) = 0$ 2) $dy/dt = +\mu/_T\, e^{-t/T} \Rightarrow \frac{dy}{dt}(0) = \mu/_T$

3) $y(T) = \mu(1 - e^{-1}) \simeq \mu \cdot 0.63$ 4) $G(0) = \mu$

EX 3.1

a) The mass balance equation is

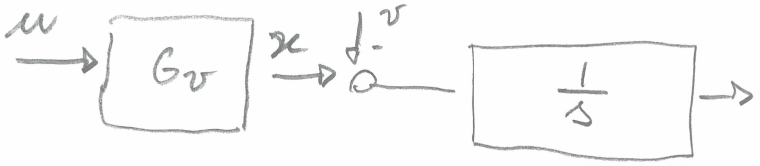
$\dot{y}(t) = x(t) - v(t)$, Apply Laplace Transform and notice that the initial conditions are 0

$sY(s) = X(s) - V(s)$, $(Y(s) = \mathcal{L}\{y(t)\},\ X(s) = \mathcal{L}\{x(t)\},\ V(s) = \mathcal{L}\{v(t)\})$

$\Rightarrow Y(s) = \frac{1}{s}(X(s) - V(s))$ BLOCK DIAGRAM

We know, from the exercise, that the transfer function between u and x is $G_v \Rightarrow X(s) = G_v(s)\,U(s)$, where $u(t)$ is the output of a PID controller.

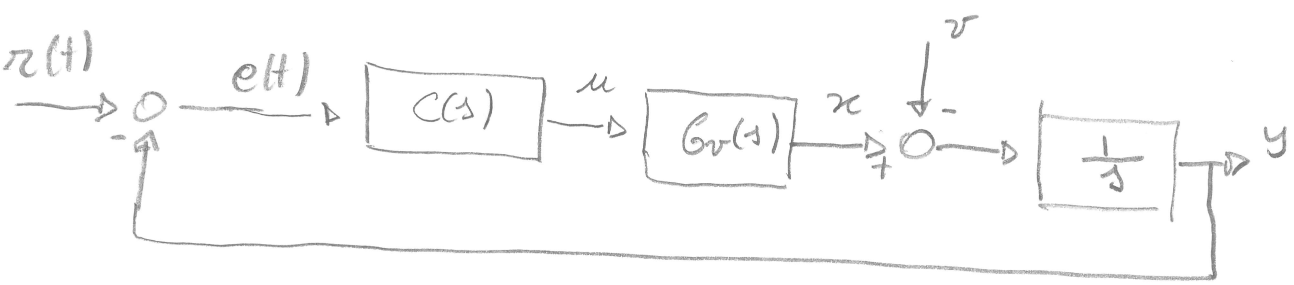
$U(s) = \mathcal{L}\{u(t)\}$



Denote by $C(s)$ the transfer function of the PID controller from $(r(t) - y(t)) = e(t)$ (the error) to $u(t)$:

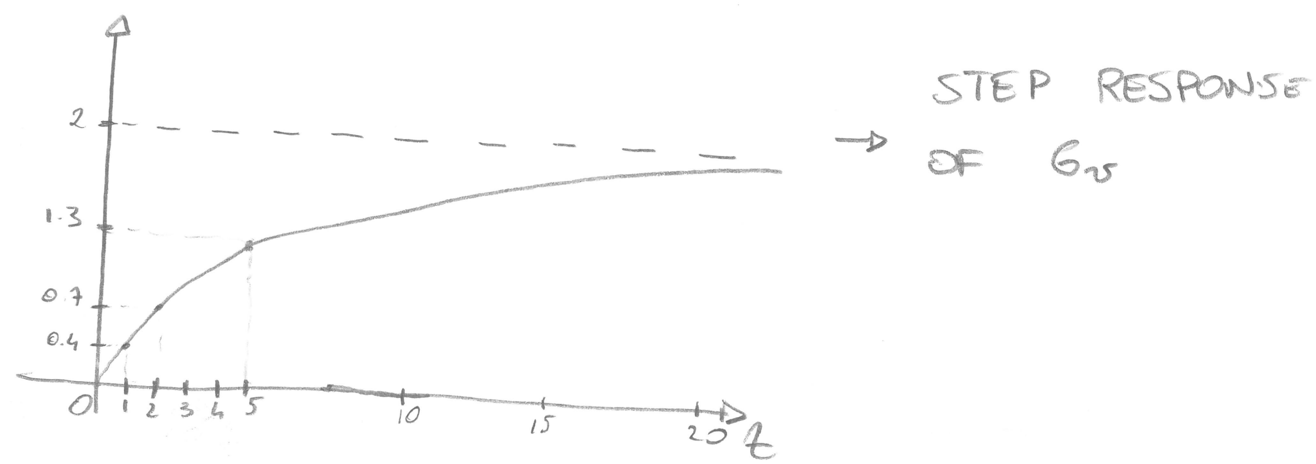
$U(s) = C(s)(R(s) - Y(s))$ $(R(s) = \mathcal{L}\{r(t)\})$

The final block diagram looks like



b

$$G_v(s) = \frac{K_v}{1+Ts}$$



Converges to 2 $\Rightarrow$ $G(0)=2$ from the final value theorem.

$G(0) = K_v = 2.$

We know that $y(T) = K_v(1-e^{-1}) = 2(1-e^{-1}) \simeq 2 \cdot 0.63 = 1.26$

$\Rightarrow T \simeq 5.$

$$G_v(s) = \frac{2}{1+5s}$$

c

To compute a transfer function from a certain input, you need to set to 0 the other inputs to that system.

$\Rightarrow$ To find transfer function between r and y we set $v=0$.

$$Y(s) = \frac{1}{s}(X(s) - \underset{0}{\underset{\shortparallel}{\cancel{V(s)}}}) = \frac{1}{s}X(s) = \frac{1}{s}G_v(s)U(s)$$

$$= \frac{1}{s}G_v(s)C(s)[R(s) - Y(s)]$$

$$Y(s)\left(1+\frac{1}{s}G_r(s)C(s)\right)=\frac{1}{s}G_r(s)C(s)R(s)$$

$$\frac{Y(s)}{R(s)}=\underbrace{\frac{1/s\,G_r(s)C(s)}{1+\frac{1}{s}G_r(s)C(s)}}_{\rightarrow \text{T.F between R and y}}$$

• Between v and y we set $r=0$

$$Y(s)=\frac{1}{s}(X(s)-V(s))=\frac{1}{s}\left(G_v(s)U(s)-V(s)\right)$$

$$=\frac{1}{s}\left(G_r(s)C(s)[\underbrace{R(s)}_{=0}-Y(s)]-V(s)\right)$$

$$=\frac{1}{s}\left(-G_r(s)C(s)Y(s)-V(s)\right)$$

$$\Rightarrow Y(s)\left(1+\frac{1}{s}G_r(s)C(s)\right)=-\frac{1}{s}V(s)$$

$$\Rightarrow \frac{Y(s)}{V(s)}=\underbrace{\frac{-1/s}{1+\frac{1}{s}G_r(s)C(s)}}_{\rightarrow \text{T.F. between V and y}}$$

Without loss of generality assume $C(s)=\frac{N_c(s)}{D_c(s)}$

$$1+\frac{1}{s}G_r(s)C(s)=\frac{s+\frac{2}{1+5s}C(s)}{s}=\frac{s+5s^2+2C(s)}{s(1+5s)}$$

$$\Rightarrow Y/V=-\frac{1+5s}{5s^2+s+2C(s)}\,,\qquad Y/R=\frac{(1+5s)G_r(s)C(s)}{5s^2+s+2C(s)}$$

$$=\frac{2C(s)}{5s^2+s+2C(s)}$$

It looks like they have same poles, but it depends on $\underline{C(s)}$!!!

Suppose $C(s) = \frac{s}{D_c(s)}$

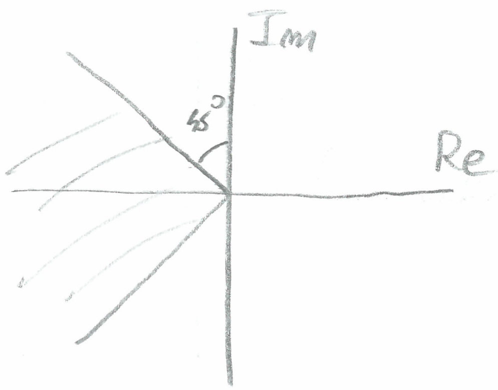
$$Y/V = -\frac{1+5s}{5s^2+s+\frac{2s}{D_c(s)}} = -\frac{(1+5s)D_c(s)}{(5s^2+s)D_c(s)+2s}$$

$$Y/R = \frac{2s/D_c(s)}{5s^2+s+2s/D_c(s)} = \frac{2s}{(5s^2+s)D_c(s)+2s} = \frac{2}{(5s+1)D_c(s)+2}$$

$\boxed{D}$

$C(s) = K \rightarrow$ for this type of controller we have same poles from V and R.

$Y/R = \frac{2K}{5s^2+s+2K}$, the poles are given by $5s^2+s+2K=0$



$$s_{1,2} = \frac{-1 \pm \sqrt{1-4\cdot 5\cdot 2K}}{2\cdot 5} = \frac{-1 \pm \sqrt{1-40K}}{10}$$

We need the real part > imaginary part (in absolute value)

$s_{1,2} = -\frac{1}{10} \pm i/_{10}\sqrt{40K-1} \Rightarrow \frac{1}{10} > \frac{1}{10}\sqrt{40K-1}$

$\Rightarrow 1 > \sqrt{40K-1}$, check when $40K-1=1$

$\Rightarrow 40K=2 \Rightarrow K = 1/20 = 0.05$

$\Rightarrow$ For $K < 0.05$ the poles are in the shadowed area.

E The disturbance signal V is a step $\Rightarrow$ $V(s) = 1/s$.

We need to check $e(t)$ at steady state

$$e(t) = r(t) - y(t) \Rightarrow E(s) = R(s) - Y(s)$$

$$Y(s) = \frac{(1+5s)G_r(s)C(s)}{5s^2+s+2C(s)}R(s) - \frac{(1+5s)}{5s^2+s+2C(s)}\underbrace{V(s)}_{=1/s} \qquad (C(s) = K)$$

Without loss of generality assume $R=0$.

$$E(s) = +\frac{(1+5s)}{5s^2+s+2K}\frac{1}{s}$$, we apply the final value thm.

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{+(1+5s)}{5s^2+s+2K} = \frac{1}{2K} \leftarrow \text{ERROR due to } V.$$

F

$$C(s) = K_p + \frac{K_I}{s} = \frac{sK_p + K_I}{s}$$

Hence $$Y/V = -\frac{(1+5s)}{5s^2+s+\frac{2sK_p+2K_I}{s}} = \frac{-(1+5s)s}{5s^3+s^2+2sK_p+K_I}$$

$$E(s) = -Y(s)V(s) = -Y(s)\frac{1}{s} = \frac{(1+5s)}{5s^3+s^2+2sK_p+K_I}$$

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{s(1+5s)}{5s^3+s^2+2sK_p+K_I} = 0.$$